Activity 3: Penalties - Teachers' notes

OVERVIEW

In this activity, the team coach invites pupils to investigate the best place to aim at to score a penalty. There's a **video** with a series of real penalty kicks that pupils can study and annotate, an **interactive penalty taking tool** for comparing theoretical and experimental probability, and a **diagram** of the penalty area and goalmouth so pupils can consider the geometry of the situation.

National Curriculum levels

This activity is aimed at those working at NC levels 4 to 7 and beyond.

Before the activity:

Preview the activity and the teachers' notes and print out the necessary pupil handouts.

Organise pupils' access to the software on PCs and teacher access via a digital projector and speakers. There is actuality sound on the video clips the pupils access during the activity, though it is not essential. However, if pupils want to replay the team coach's introductory video clips themselves, they will need sound.

Decide which elements of the activity will be 'whole class' and how pupils will work in small groups (suggestion of groups of 2-3 pupils around each computer).

During the activity:

Show the class the team coach's introductory video to the whole case study.
[This could be skipped if the class is familiar with it from a previous activity.]

"Hello! I'm Joe Bailey, the coach of Swanscombe Tigers football club, and you're going use mathematics to analyse the performance of my team. In these activities I want you to look at how they train, how they pass the ball and how they score goals. So choose a topic and report back to me with what you find out."

Select the activity to be investigated.

Show the class the football coach's introductory video to this activity.

"Penalty kicks are very important - they can turn a game - so players need to think hard about where to aim at to score a goal. I want you to look at some video clips of my players taking penalty kicks and report back to me with your ideas about the best place to aim at - and why. Is the probability of scoring a goal different for the different zones of the goalmouth? Does it depend on the speed of the kick, or the goalkeeper's reaction time, or how far he can reach? Or is it just a question of luck! You tell me!"

Start pupils working on the activity and provide support to groups and individuals.

Organise the whole class plenary for pupils to evaluate and feedback on their work at the end of the activity.

After the activity:

Consider whether there are opportunities for pupils to replicate this activity for themselves, such as in PE or games lessons.

Supporting documentation for teachers

- **Teachers' Notes** with an overview of the activity and a suggested lesson plan.
- **Extension ideas** with suggestions for further work using the same content

Supporting documentation for pupils

- **Interactive penalty taking worksheet** with suggestions about how to use this probability activity and to log the results.
- **Penalty geometry worksheet** with a 3-D diagram of the penalty area and goalmouth
- **Pupil Report Sheet** for pupils to record their ideas about what factors contribute to a successful penalty kick and to report back to the team coach.

Resources required

- A computer and projector with speakers for the teacher (essential)
- A computer for each group of 2 or 3 pupils (recommended)
  …with speakers or headphones (optional)
- A copy of the *Interactive penalty taking worksheet* per group (recommended)

and/or

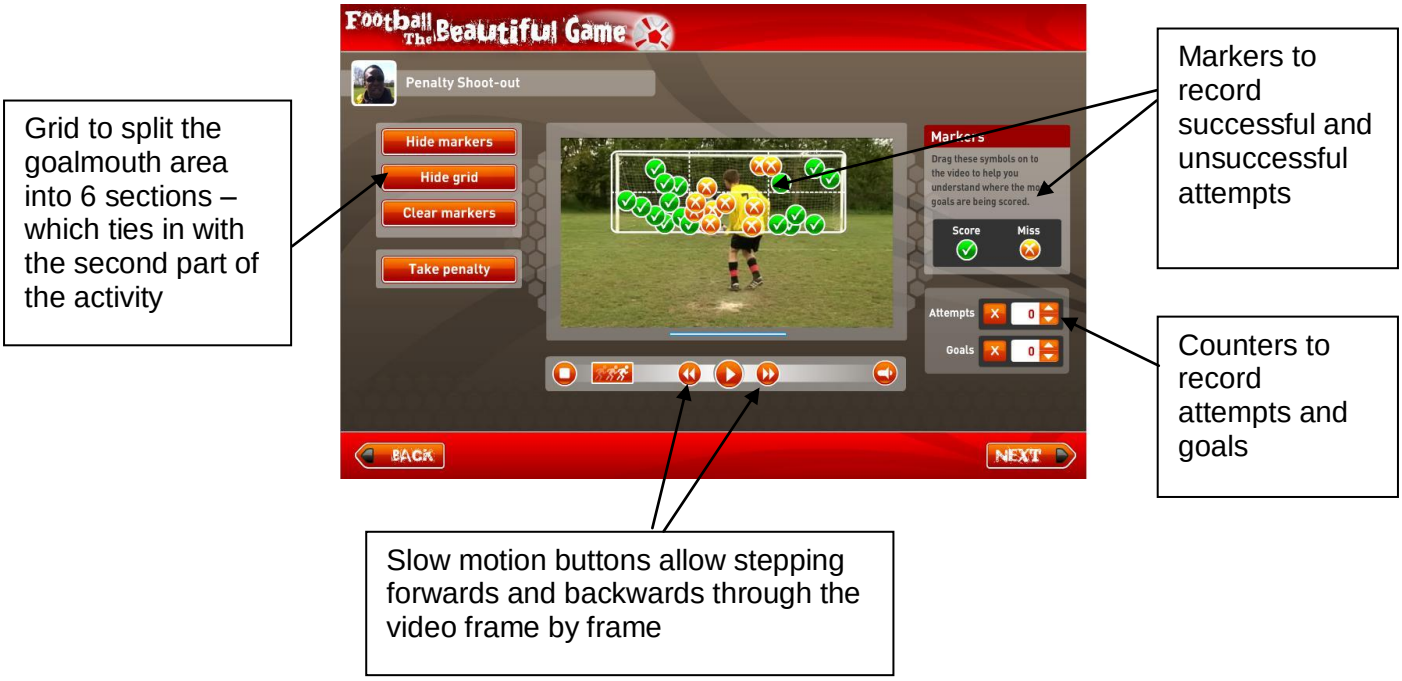
- A copy of the *Penalty geometry worksheet* per group (recommended)
- A copy of the *Pupil Report Sheet* per group (recommended)

LESSON PLAN

Start of the lesson (5 minutes)

After playing the introductory videos, establish that the pupils understand the task the football coach has set them - to consider the best places to aim at to score a goal.

Show a few sample penalty kicks from the video and ask the class for their initial thoughts, taking feedback from a small number of pupils to stimulate their thinking. Demonstrate the grid, which divides the goal into six 'zones', and the tools pupils can use to mark the location of goals and saves. Then ask them to watch a few more video clips to see what pattern emerges.



Pupils' initial exploration and feedback (10-15 minutes)

Pupils work in groups of two or three to annotate the video penalty kicks and come up with ideas as to which of the six 'zones' seem most successful and why. In the video there are about 40 separate kicks, some fast and some slow, some saved and some not.
It will be useful for pupils to save their final pattern of goals and saves by using 'Print Screen' on their keyboard and pasting it into a blank 'Word' document so they can refer to it during the second activity. Also, beware the different effect of 'Hide Markers', which is temporary, and 'Clear Markers', which is permanent.

Lead a class discussion about what pupils have noticed and record their thoughts on the board, which might include what variables they think are involved, such as distance, speed of ball, the goalkeeper's reactions - or luck!

Questions to pose:

- Is it better for the goalkeeper to dive to the right, to the left, or stand still?
- Where should the penalty taker aim?
- Assuming the goalkeeper starts in the middle of the goal, are there parts of the goal he can never reach in a penalty situation?
- With the goalmouth divided into 6 sections (see the grid on video clips) what is the chance that a shot aimed at a specific section will be successful?

NB. It is important for later work that this is completed quite thoroughly. For the purpose of this mathematical exercise, you'll need to exclude any suggested 'real life' factors, such as the goalkeeper knowing which way a particular striker usually kicks, or looking for visual clues in their run up. Mathematically simplified, there are six zones the striker can aim at and six zones the goalkeeper can move to.

Pupils' further exploration (20 - 25 mins)

Depending on the available time, you may prefer pupils to work either with the penalty taking tool or the geometrical diagram, rather than both, or reserve the *Penalty Geometry* for more able pupils.

Interactive penalty taking tool:

This tool is to help pupils understand that there is a difference between theoretical probability and experimental probability. It will help them to appreciate that, with more trials, experimental probability gets closer to the theoretical probability – this is easier to see with decimal or percentage settings than with fraction ones. This activity does need initial teacher input to ensure that pupils understand how to express probability and how the tool works.

The probabilities entered in any one zone apply only to that zone, so entering 100% into zone 1 and 0% in the other zones sets the tool so that, however unrealistically, all kicks directed towards zone 1 will be successful goals whereas all shots aimed at all other zones would be saved.

Show the pupils the screen and demonstrate how to input the settings. One suggestion is to input settings of 'zero' for the middle sections, '1' for the top corners and '0.5' for the bottom corners. The probabilities in each zone are independent of each other and so do not 'add up to one'. It is important that pupils realise that when they put a setting of '0.5' in a zone, they are setting the tool so that there is a 50% probability that any one shot aimed at that part of the goalmouth will score a goal. Shots aimed at other parts of the goalmouth will have different chances of being successful, according to the settings pupils put in.

Use the *Take a penalty* button and then *Take 10 penalties*. Show how to compare the 'actual scores' with 'your settings' by toggling the actual scores/my settings button. Ask the pupils to interpret what has happened in the zones with settings of 'zero' and '1' i.e. that none of the shots aimed at the zones with a probability setting of 'zero' was successful whereas all of the shots aimed at zones with a setting of '1' were successful in scoring a goal. *Do not provide an interpretation for the '0.5' zones at this point as this is what the pupils will be exploring and trying to make sense of for themselves.*

Point out that the number of kicks directed at each zone varies because the kicks are randomly aimed at the zones. Switching to see the scores as *Fractions* makes it easier to see how many actual shots have been aimed at each zone and how many were successful. This is not so apparent with percentage or decimal notations. There's also a running total of attempts and goals on the right hand side.

Pupils can then use the interactive penalty taking tool to input their own settings for the probability of scoring a goal aimed at each zone of the goalmouth and take a series randomly directed penalty kicks to see how closely the experimental outcome matches the values that they set. The pupil log sheet will help them to explore scenarios, pose questions and record outcomes.

Pupils can compare the settings they entered with the 'actual' proportion of goals scored. They will notice a difference between their settings and the actual values. They may even

observe different outcomes for the same entered values ie if they entered 25% in every section does this mean that all the 'actual' values turn out to be the same as each other?

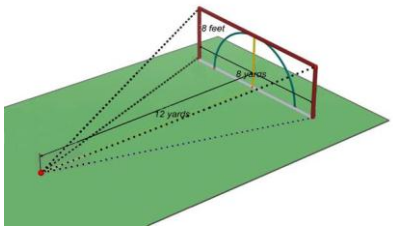
By repeatedly using *Take 100 penalties* or even *Take 1000 penalties* to take more kicks, pupils should begin to notice that their entered settings and the actual results begin to converge and stabilise and they can discuss why that might be and how many kicks it needs before it happens.

If time permits, pupils can try entering more realistic settings based on their judgment of the probability of scoring in the various zones, based on their earlier study of the actual video penalties.

The *'data reset'* button clears the accumulated goals and allows different figures to be entered. *'Save to CSV'* puts the accumulated data into clipboard from where it can be pasted into a spreadsheet.

Encourage discussion about the difference between theoretical and experimental probability

Penalty geometry:



This is a video 3-D diagram of the penalty area, based on an interactive Cabri file, which pupils can move around and look at from different angles. You can download the cabri software that created this from http://www.cabri.com/cabri-3d.html. The source file from the video is called Goalmouth.cg3 and can be found in the folder named 'cabri' on the DVD version of the activities. There's a paper version on their worksheet. The video simply gives the pupils the chance to see the layout from different perspectives. Pupils are either proficient at visualising a 3-dimensional situation from a 2-dimensional diagram, or they're not. Those who are not proficient often struggle to understand that there are right angled triangles in a vertical plane. This animation is designed specifically to help them see that. It may be worth grouping pupils so that 'poor visualisers' are paired with 'good visualisers' who will be better able to help them interpret and understand.

The dimensions are provided, with a semi-circle suggesting how far the goalkeeper might reach from the centre of the goal line. This is, however, a very rudimentary locus since when standing still the goalkeeper will be able to reach higher than he/she will be able to reach sideways.

On the pupil sheet, they are told that the speed of a strong penalty kick is about 60 mph (88 feet/sec) and that the goalkeeper's reaction time is about a quarter of a second.

Pupils are asked to use this information to work out which parts of the goal might theoretically be easier to score in and why. They can do this by finding the distances to the different parts of the goal - by estimating, constructing similar triangles or calculating using Pythgoras' theorem - and then working out how long the ball takes to get there.

Pupils may need supporting in their understanding of the interrelationship between speed, time and distance, which is a major factor in this activity and will also will need an understanding of imperial units of measure. They may have used the 'speed, distance, time triangle' in science lessons or could be encouraged to think about the interrelationship from first principles by considering how long it would take someone to travel 20km if they: dawdle at 2km/h, or walk at 4km/h, or cycle at 10km/h or 20km/h, or drive at 40km/h.

They will need to consider how far the goalkeeper can move in the time it takes for the ball to reach various parts of the goalmouth. Although some parts - the top corners - are further away from the goalkeeper's starting position, it will take the ball longer to get there which gives the goalkeeper a bit more time. For a fast kick, this difference in time is very small, so pupils are asked to also consider how much difference a slower kick would make.

Questions that could be posed to pupils who struggle to find an approach to solve this problem:

- What are the different distances to the different parts of the goal?
- How long does the ball take to get there?
- How far can the goalkeeper move in that time?

Pupils need to be aware of the various units employed here. The Rules of Football define the dimensions of the penalty area and the goal in Imperial measures, so pupils may need reminding that 12 inches = 1 foot, 3 feet = 1 yard, and 1760 yards = 1 mile

If pupils prefer to work in metric measures they will need to convert the dimensions
1 yard = 0.914metres and 1 mile = 1.61km

Plenary (10-15 minutes)

Towards the end of this session you may wish to run the coach's 'reminder' video.

"Now you've got all the information you need, it's time to report back to me with your theories about the best way to score a penalty."

Groups complete and feed back their responses and justification.

In answer to the team coach's original question:

- The harder and quicker the kick, the less chance the goalkeeper has of being able to react and move in time to save it.
- The two top corners of the goal are the most successful places to aim at as they are the furthest for the ball to get to, but they're also furthest for the goalkeeper to reach.
- The two bottom corners of the goal are the next most successful places to aim at as they're the next most difficult for the goalkeeper to reach.
- Because of their reaction time, it's too late for the goalkeeper to wait until he sees which way the ball's been kicked and then move to save it. In practice they have to guess and try to anticipate where the kick is going to go to stand any chance of saving it.
- Aiming at the centre of the goal can be successful as there's a possibility that the goalkeeper will move, thinking the ball is going into one of the corners.
- Many strikers decide in advance where they're going to aim for, irrespective of what the goalkeeper does, as if they change their mind at the last minute, research shows their kick becomes a lot less accurate.
- Mathematically simplified, the chances of scoring a penalty are the result of the probabilities involved with six zones the striker can aim at and six zones the goalkeeper can move to. Where these coincide, it's more likely to be a save, and where they don't it's a goal.
- So it looks like scoring a penalty is a combination of speed, accuracy – and luck!

Lead a whole class discussion about any difficulties pupils encountered using real data.

How confident are pupils about the decisions they have made? This would be useful to help pupils understand the limitations of their work. What might they have gone on to investigate if they had more time?

KEY PROCESSES AND CONTENT

Curriculum opportunities

During the key stage pupils should be offered the following opportunities, which are integral to their learning and enhance their engagement with the concepts, processes and content of the subject.

The curriculum should provide opportunities for pupils to:

• work on sequences of tasks that involve using the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts

• work on open and closed tasks in a variety of real and abstract contexts that allow pupils to select the mathematics to use

• work on problems that arise in other subjects and in contexts beyond the school

• work on tasks that bring together different aspects of mathematical content, involving use of several of the key processes, or require using the handling data cycle

• work collaboratively as well as independently to solve mathematical problems in a range of contexts, evaluating their own and others' work and responding constructively

• use a variety of resources when solving problems or carrying out mathematical procedures.

Key processes

Representing

Pupils should be able to:

• identify the mathematical aspects of the situation or problem

• choose between representations

• simplify the situation or problem in order to represent it mathematically using appropriate variables, symbols, diagrams and models

• select mathematical information, methods and tools to use.

Analysing

Use mathematical reasoning

Pupils should be able to:

• make connections within mathematics

• use knowledge of related problems

• visualise and work with dynamic images

• look for and examine patterns and classify

• make and begin to justify conjectures and generalisations, considering special cases and counter examples

• explore the effects of varying values and look for invariance

• take account of feedback and learn from mistakes

• work logically towards results and solutions, recognising the impact of constraints and assumptions

• appreciate that there are a number of different techniques that can be used to analyse a situation

• reason inductively and deduce

Use appropriate mathematical procedures

Pupils should be able to:

• make accurate mathematical diagrams, graphs and constructions on paper and on screen

• calculate accurately, using a calculator when appropriate

• manipulate numbers, algebraic expressions and equations and apply routine algorithms

• use accurate notation, including correct syntax when using ICT

• record methods, solutions and conclusions

• estimate, approximate and check working.

Interpreting and evaluating

Pupils should be able to:

• form convincing arguments based on findings and make general statements

• consider the assumptions made and the appropriateness and accuracy of results and conclusions

• be aware of strength of empirical evidence and appreciate the difference between evidence and proof

• look at data to find patterns and exceptions

• relate findings to the original context, identifying whether they support or refute conjectures

• engage with someone else's mathematical reasoning in the context of a problem or particular situation

• consider whether alternative strategies may have helped or been better.

Communicating and reflecting

Pupils should be able to:

• communicate findings in a range of forms

• engage in mathematical discussion of results

• consider the elegance and efficiency of alternative solutions

• look for equivalence in relation to both the different approaches to the problem and different problems with similar structures

• make connections between the current situation and outcomes, and ones they have met before.

Curriculum content

Number and algebra

• rational numbers and their different representations

• rules of arithmetic applied to calculations and manipulations with rational numbers

• applications of ratio and proportion

• accuracy and rounding

• algebraic expressions, formulae, equations, inequalities and identities including index notation and the use of brackets to indicate precedence

• simultaneous linear equations in algebraic and graphical forms

• sequences, including those arising from rules, in a variety of contexts

• graphs of polynomial functions and their properties

Geometry and measures

• properties of 2D and 3D shapes and their applications, including constructions, loci and bearings, deductive reasoning and Pythagoras' theorem

• transformations, similarity and congruence including the use of scale

• points, lines and shapes in 2D coordinate systems

• units, compound measures and conversions

• perimeters, areas, surface areas and volumes

Statistics

• presentation and analysis of grouped and ungrouped data including time series and lines of best fit

• measures of central tendency and spread

• experimental and theoretical probabilities including those based on equally likely outcomes

• applying statistics to enable comparisons.